1.(30%) The height of an ocean wave is described by the function

$$y(x, t) = 1.5\sin(0.5t - 0.6x) \text{ (m)}.$$

Determine the phase velocity (10%) and the wavelength (10%), and then sketch $y(x, t)$ at t = 2s over the range from $x = 0$ to $x = 2\lambda$.

Solution: The given wave may be rewritten as a cosine function:

$$y(x,t) = 1.5\cos(0.5t - 0.6x - \pi/2).$$

By comparison of this wave with Eq. (1.32),

$$y(x,t) = A\cos(\omega t - \beta x + \phi_0),$$

we deduce that

$$\omega = 2\pi f = 0.5 \text{ rad/s}, \qquad \beta = \frac{2\pi}{\lambda} = 0.6 \text{ rad/m},$$

10% $$u_p = \frac{\omega}{\beta} = \frac{0.5}{0.6} = 0.83 \text{ m/s}, \qquad \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.6} = 10.47 \text{ m}.$$ 10%

At $t = 2$ s, $y(x,2) = 1.5\sin(1 - 0.6x)$ (m), with the argument of the cosine function given in radians. Plot is shown in Fig. .

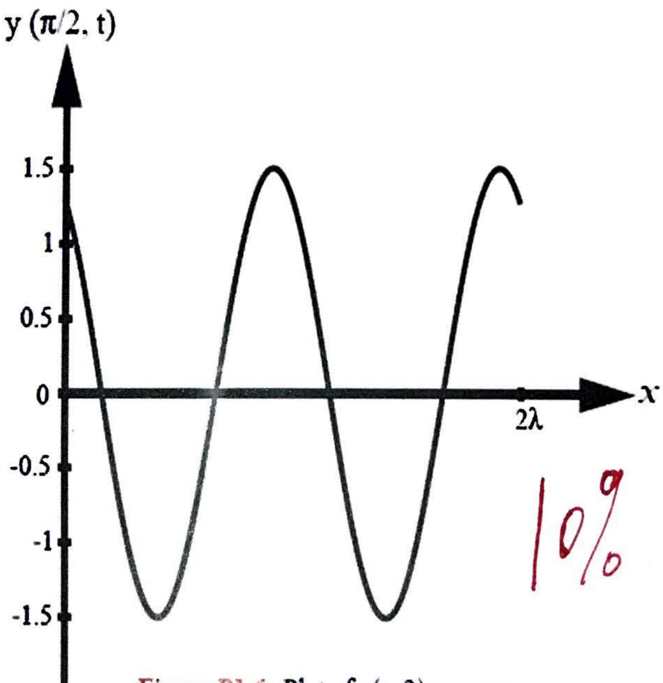

Figure P1.6: Plot of $y(x, 2)$ versus x.

2. (30%) An air spaced lossless 50-Ω line ($\varepsilon_r = 1$) is terminated in a load with impedance of $Z_L = 60 + j60$-Ω at frequency 5GHz, Find (1) the reflection coefficient (10%); (2) the voltage standing wave ratio (S) (10%) and (3) the location of the first voltage maximum from the load in centimeters (10%).

Solution:

(1) $$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{60 + j60 - 50}{60 + j60 + 50} = 0.49\angle 0.91$$ 5%

(2) $$S = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.49}{1-0.49} = 2.9$$ 5%

(3) $$l_{max} = \frac{\theta_r}{4\pi}\lambda = 0.072\lambda$$

$$\lambda = \frac{c}{f} = \frac{3\times10^8 m/s}{5\times10^9 Hz} = 6cm$$

$$l_{max} = 0.43cm$$ 5%

2. (30%) An air spaced lossless 50-Ω line ($\varepsilon_r = 1$) is terminated in a load with impedance of Z_L = 60 + j60-Ω at frequency 5GHz, Find (1) the reflection coefficient (10%); (2) the voltage standing wave ratio (S) (10%) and (3) the location of the first voltage maximum from the load in centimeters (10%).

Solution:

(1) $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{60 + j60 - 50}{60 + j60 + 50} = 0.49\angle 0.91$. 5%

(2) $S = \dfrac{1+|\Gamma|}{1-|\Gamma|} = \dfrac{1+0.49}{1-0.49} = 2.9$ 5%

(3) $l_{\max} = \dfrac{\theta_r}{4\pi}\lambda = 0.072\lambda$.

$\lambda = \dfrac{c}{f} = \dfrac{3\times10^8 m/s}{5\times10^9 Hz} = 6cm$

$l_{\max} = 0.43cm$. 5%